

# BIOE: Biostatistics Course Fall 2017

## Assignment 3

**Due 26th October**

For plotting use Matplotlib and provide the code you used with the assignment.

### COUNTING

1. How many lunches are possible consisting of a soup, a sandwich, a desert and a drink if one can select from four different soups, three kinds of sandwiches, five deserts and four drinks?

$$4 \cdot 3 \cdot 5 \cdot 4 = 240$$

2. In tossing 7 coins what is the **total** number of arrangements of heads and tails?

$$2^7 = 128$$

3. How many **permutations** are there for the word ORANGE?

$$6! = 720$$

4. How many **combinations** are there for the word ORANGE?

$$1$$

5. How many ways is it possible to arrange 7 coins made up of 4 heads and 3 tails?

$$C(7,4) \text{ or } C(7,3) = 35$$

6. How many different 5-letter arrangements are there in the word MOOSE?

$$60$$

7. In the production of the new Blade Runner movie 2049, eight actors were considered for the male roles of 'K', Wallace, and Galf. In how many ways could the director cast the male roles?

336, permutations.

8. A password consists of three digits, 0 through 9, followed by three letters from an alphabet having 26 letters. If repetition of the digits is allowed, but repetition

of the letters is not allowed, determine the number of different passwords that can be made.

$$10 \cdot 10 \cdot 10 \cdot 26 \cdot 25 \cdot 24 = 15,600,000$$

9. A customer selects three different toppings for a pizza. If there are 9 different toppings from which to choose, how many different pizzas can be made?

$${}^9C_3 = 84$$

10. Determine the values for each of the following:

$$\binom{7}{3}, \quad \binom{5}{2}, \quad \binom{8}{4}, \quad \binom{6}{0}, \quad \binom{9}{9}$$

35, 10, 70, 1, 9

## DISTRIBUTIONS

11. Suppose you have a key ring with eight keys, one of which is your house key. Suppose that you get home after dark and you can't see the keys. You randomly try one key at a time making sure you never try a key more than once. What is the probability of that you get the right key?

a) On the first try?

$$1/8$$

b) Eighth try?

$$7/8 \times 7/8 \times 7/8 \times 7/8 \times 7/8 \times 7/8 \times 7/8 \times 1/8$$

c) Before the fourth try?

$$1/8 + (7/8 \times 1/8) + (7/8 \times 7/8 \times 1/8) \quad 12. \text{ In a large restaurant, an average of 3}$$

out of every 5 customers asks for wine with their meal. A random sample of 10 customers is selected

a) What is the probability that 6 customers asked for wine?

$$\text{approx } 0.6161$$

b) What is the probability that no customers asked for wine?

$$0.000105$$

c) What is the probability that 1 or 2 customers asked for wine?

$$0.0122$$

d) What is the probability that all 10 customers asked for wine?  
0.00061

13. Write a simple Python program to help you compute a probability from a binomial distribution, given the number of trials, and probability of success. Provide examples of the program in operation.

14. A lab technician has to transfect a series of cell cultures containing the same cell type and growth media. The probability of a successful transfection is 0.15.

a) Justify that the success or failure of the technician's experiment can be described using a binomial distribution.

Independent, fixed number of trials and constant probability between trials

b) What is the probability that the technician ends up with no cell cultures transfected if they manage to try to transfect 10 cultures in a single day?

0.197

c) What is the mean number of cell cultures that will be transfected if the technician tries to transfect 10 cell cultures?

$np = 10 * 0.15 = 1.5$

d) What is the probability that at least 3 cell cultures will be transfected if the technician manages to try to transfect 20 cell cultures in a single day?

= 0.352

e) The lab technician gets a bonus of \$100 if they manage to transfect all 10 cell cultures. What is the probability that the lab technician will get the \$100? Do you think this would be a good way to get rich?

5.7665039062500008e-09 //

15. A manufacturer produces artery stents of which 1% are defective. Stents are packaged in boxes of 10 and sent to hospitals. A box is selected at random.

a) Find the probability that the box contains exactly one defective component.

0.0914

b) Find the probability there are at **least** 2 defective stents in the box

$P(X \geq 2) = 0.0043$

c) Find the probability that a batch of 250 stents contains between 1 and 4 (inclusive) defective stents. Hint: You probably won't be able to do this problem using your calculator because the factorial for 250 will exceed your calculator's bounds.

You could solve it by approximating the Binomial as a Poisson because  $n$  is so large. **However** for this problem use Python to compute the binomial distribution in order to help you solve the problem.

```
0.811 import scipy import scipy.stats as ss hh = ss.binom(250, 0.01) print (hh.pmf (1) + hh.pmf (2) + hh.pmf (3) + hh.pmf (4))
```

16. The mean of a binomial distribution is 2.5 and was calculated from a set of 10 independent Bernoulli trials. What is the probability of success in a single trial?  
 $p = \text{mean}/n = 0.25$

17. From question 16, what is the variance and standard deviation of the binomial distribution that had mean 2.5?  
 $\text{var} = n p (1-p) = 10 \times 0.25 (1-0.25) = 2.5 \times 0.75 = 1.875$   $\text{sd} = \text{sqrt} (1.875)$

18. Patients arrive at a hospital accident and emergency department at random at a rate of 6 per hour.

a) Justify that the distribution of patients entering the hospital can be described as a Poisson distribution.

Find the probability that during any **90 minute period**, the number of patients arriving at the hospital accident and emergency department is:

b) Exactly 7

$\text{Lambda} = 9$  for a 90 min period ( $1.5 * 6$ ) 0.117 c) At least 10

$1 - P(X \leq 9) = 0.413$

d) A patient arrives at 11.30am, what is the probability that the next patient arrives before 11.45 am?

Set  $\text{lambda}$  to 1.5 which is the number of patients arriving per 15 mins.  $P(X \leq 1) = 1 - P(X = 0) = 0.777 /$

19. A researcher is studying the distribution of fibroblasts on a new biomaterial. The material has been marked using a microlaser so that it is divided into a number of equally sized squares. The mean number of fibroblasts per square is found to be 3 and it is assumed the cells are distributed randomly, no clumping or regular patterns were observed. Given a randomly chosen square:

a) What is the probability that there will be more than 2 fibroblasts

b) What is the probability that there will be either 5 or 6 fibroblasts

The researcher decided to count all the fibroblasts in each of 80 randomly selected squares. The total number of cells was found to be 295. The researcher also computed the sum of the squares, ie  $\sum x_i^2 = 1386$  where  $x_i$  is the number of cells found in the  $i$ th square.

c) From the above data compute the mean number cells per square for the 80 squares

$295/80 = 3.69$  d) From the above data compute the variance of the number of cells

per square for the 80 squares

3.727

e) Use the information about the mean and variance to support the assertion that the Poisson distribution is a good model for the distribution of the fibroblasts and hence supports the notion that the fibroblasts are distributed randomly.

Mean and variance are roughly equal, suggesting poisson model /

f) Using the mean computed in c), estimate the probability that exactly 4 fibroblasts will be found in a randomly selected square.

insert into Poisson formula, 0.193