Assignment 2

Due Oct. 19

1.

b.

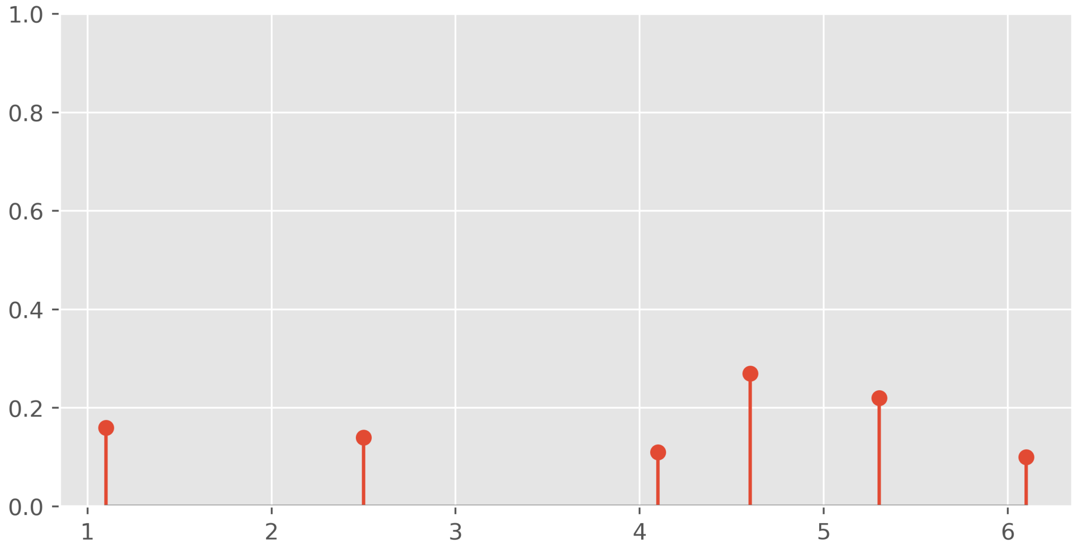
2.

b.

3.

Probability of random variable X get the value of x.

4.



5.

You can't have negative values for probability.

PMF for a probability distribution should sum up to 1.

6.

a) 0.25.

b) 0.53

c) 0.47

d) $\mu = \sum_i x_i p(x_i) = 19.76$

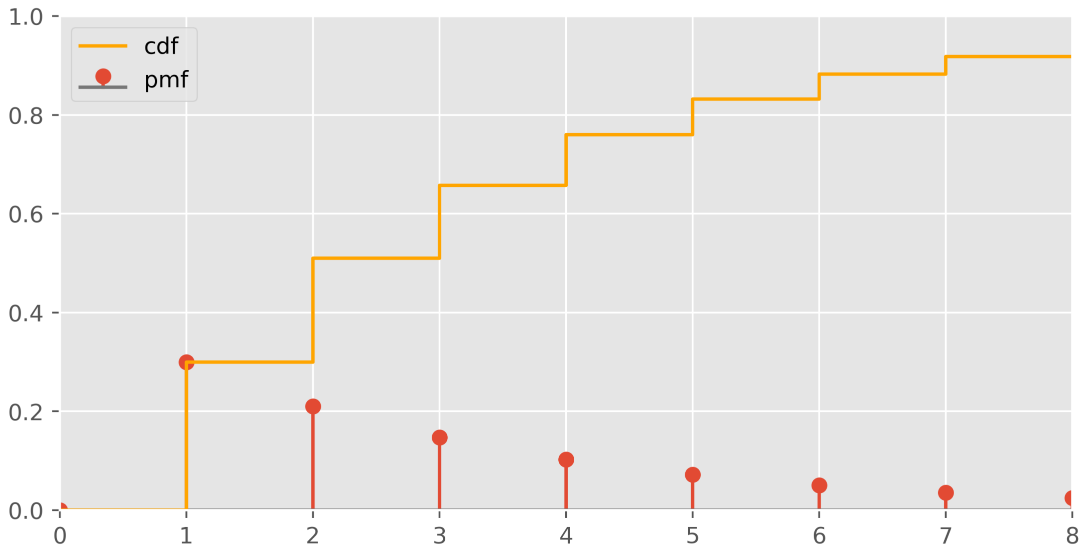
e)$\sigma^2 = \sum_i p_i x_i^2 - \mu^2 = 22.2824$

f)$\sigma = 4.7204$

7.

[0.3, 0.51, 0.657, 0.76, 0.832, 0.8824, 0.9178, 0.9428]

8.



9.

Denote 1-q as p,

$$E(X) = \sum_k^\infty k \cdot p^{k-1} \cdot q$$
$$= (1 + 2p + 3p^2 + \ldots + kp^{k-1} + \ldots)q$$

Denote,

$$S_k = 1 + 2p + 3p^2 + \ldots + kp^{k-1} + \ldots \quad (1)$$
$$pS_k = p + 2p^2 + 3p^3 + \ldots + kp^k + \ldots \quad (2)$$

$(1) - (2)$ we have,

$$(1-p)S_k = 1 + p + p^2 + \ldots + p^{k-1} - kp^k$$
$$S_k = \frac{1-p^k}{(1-p)^2} - \frac{kp^k}{1-p}$$

As p is a probability, $0 \le p \le 1$. Thus, $\lim_{k \Rightarrow \infty} kp^k = 0, \lim_{k \Rightarrow \infty} p^k = 0$.

$$S_k = \frac{1}{(1-p)^2} = \frac{1}{q^2}$$
$$E(X) = \frac{1}{q}$$

10.

a)

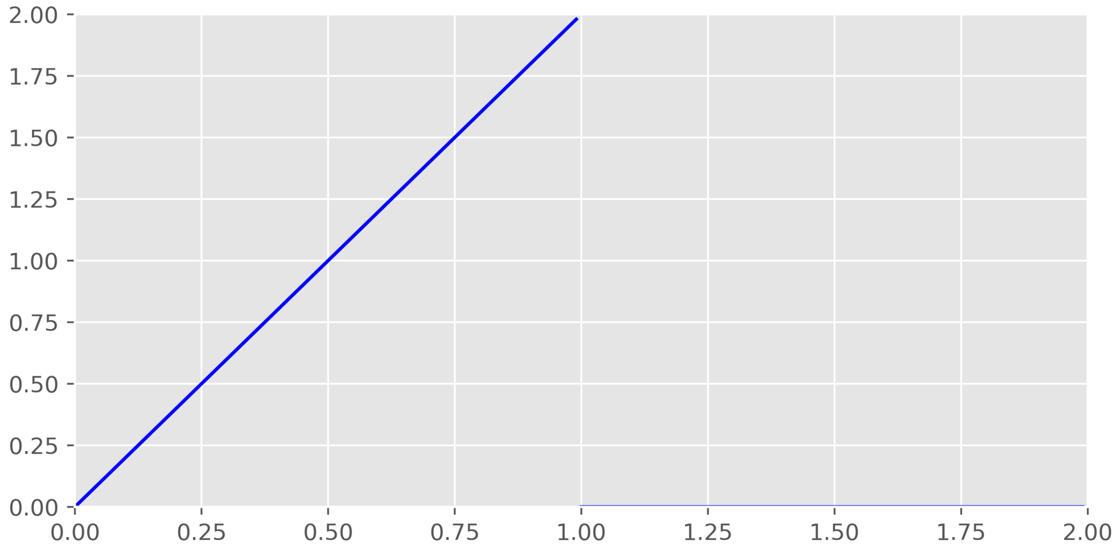
$P(k=4) = (1-p)^4 p = (1-0.05)^4 \times 0.05 = 0.0407253125$

b)

$E(X) = \frac{1}{p} = 20$

11.

a)



b)

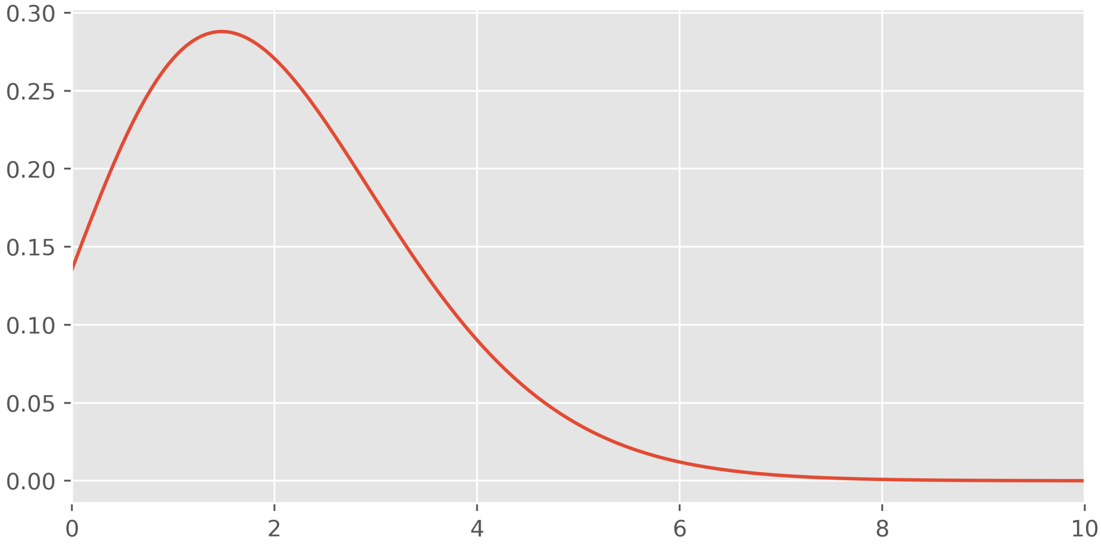
$$P(0 \leq X \leq 0.5) = \int_0^{0.5} f(x) \cdot dx = 1/4$$

c)

$$P(1/4 \leq X \leq 3/4) = \int_{1/4}^{3/4} f(x) \cdot dx = 1/2$$

12.

a)



b)

$$p(k=5) = e^{-2}\frac{2^5}{5!} = 0.036$$

13.

```python
from __future__ import division
import numpy as np

def genVariance(a, size):
    '''
    Here is a function gen variance of sample of a normally distributed
    variable X and aX with 0 mean and unit varianve with given size and a.
    '''
    #gen random variable X with 0 mean and unit variance sample size 1000
    X = np.random.normal(0, 1, size)
    #calculate variance
    vari = np.var(X)
    #calculate var of aX
    vari_ = np.var(a * X)

    return vari, vari_

sim_size = 1000 #define simulation times

varis = [] #variances for X
avaris = [] #variances for aX
a = 3 #define a
for i in range(sim_size):
    vx, vax = genVariance(a, 1000)
    varis.append(vx)
    avaris.append(vax)

print('var(aX) \ var(X) = ' + str(np.average(avaris)/np.average(varis))
               + ', where a = ' + str(a))
```

14.

a)

$$\frac{df(x)}{dx} = -2$$
$$\underset{x}{argmin}\, f(x) = 0, x = 1$$

b)

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0\ dx + \int_{0}^{2} 2(1-x)dx + \int_{2}^{\infty} 0\ dx = 1$$

c)

$$\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} 0\ dx + \int_{0}^{2} 2x(1-x)dx + \int_{2}^{\infty} 0\ dx$$
$$= \frac{1}{3}$$

d)

$$\begin{aligned}\sqrt{\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx} &= \sqrt{\int_{-\infty}^{0} 0\;dx + \int_{0}^{2} 2(x-\mu)^2(1-x)dx + \int_{2}^{\infty} 0\;dx} \\ &= \frac{\sqrt{2}}{6}\end{aligned}$$