Continuous Distributions: Exponential[1]

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[1] HMS, 2017, v1.0

Chapter References

- Navidi, Chapter 4.7

The Exponential Distribution

- The exponential distribution is used to model **waiting times**.
- In systems biology and chemistry a common waiting time is the time it takes a single reaction to occur.

The Exponential Distribution

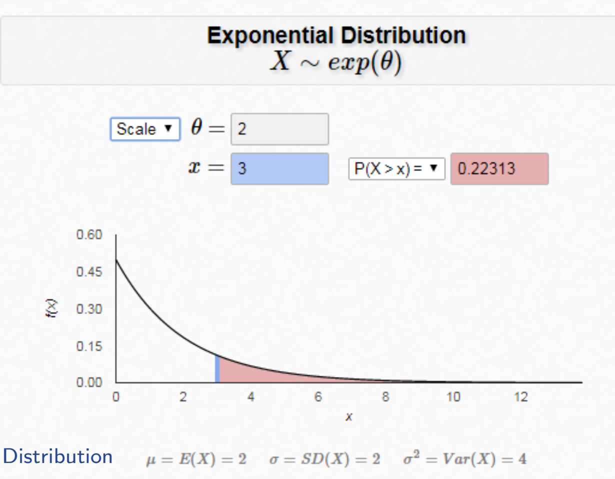
Note: None of these express events per unit something as in a Poisson distribution.

- Time for component to fail.
- Time for a cell to die.
- How long a telephone call lasts
- The time until you have your next car accident.
- The time until a radioactive particle decays, or the time between beeps of a Geiger counter.
- The number of dice rolls needed until you roll a six 11 times in a row.
- The time until a large meteor strike causes a mass extinction event.
- Distance between mutations on a DNA strand.
- The distance between roadkill on a given street.
- Time between cars passing under a bridge.

The Exponential Distribution

- The exponential distribution function involves a single positive parameter, λ.

$$f(x) = \lambda e^{-\lambda x} \qquad f(x) = 0 \text{ if } x \leq 0$$



The Exponential Distribution

$$f(x) = \lambda e^{-\lambda x} \qquad f(x) = 0 \text{ if } x \leq 0$$

$$F(x) = P(X \leq \infty) = \int_0^\infty \lambda e^{-\lambda x} dx$$

Recall:

$$\int_0^\infty e^{-ax} dx = \frac{1}{a}$$

$$F(x) = \lambda \frac{1}{\lambda} = 1$$

The Exponential Distribution: Something Useful

$$F(x) = P(X \leq x) = \int_0^x \lambda e^{-\lambda x} dx = 1 - e^{-\lambda x}$$

Useful for finding the area under an exponential curve.

The Exponential Distribution

$$f(x) = \lambda e^{-\lambda x} \qquad f(x) = 0 \text{ if } x \leq 0$$

Mean: $\mu = \frac{1}{\lambda}$

Variance: $\sigma^2 = \frac{1}{\lambda^2}$

The Exponential Distribution: Memoryless Property

What happens if the ocular implant is already 10 years old, what will the probability be that it lasts longer than an additional three years?

Exactly the same probability as if it were new.

$$P(T > 3) = 1 - P(T \leq 3) = 1 - (1 - e^{-2x}) = 0.223$$

Relationship to Poisson Distribution

If events follow a Poisson process with rate λ and if T represents the waiting time from any starting point until the next even then:

$$T \sim \text{Exp}(\lambda)$$

The number of cars passing under a bridge is Poisson distributed then the time between cars is exponentially distributed.

Relationship to Poisson Distribution: Example

A radioactive element emits particles according to a Poisson distribution at a mean rate of 15 particles per minute.

A some point a clock is started. What is the probability that more that 5 seconds will elapse before the next emission?

What is the mean waiting time until the next particle is emitted?

The mean rate of emissions os 0.25 per second (given 15 per minute). Therefore $\lambda = 0.25$ and so $T \sim \text{Exp}(0.25)$.

$$P(T > 5) = 1 - P(T \leq 5) = 1 - (1 - e^{-0.25(5)}) = 0.2865$$

The mean waiting time: $1/\lambda) = 1/0.25 = 4$ seconds.