BIOE 336 Midterm: 2012

Name:

The number in square brackets at the start of each question represents the awarded points. Total number of awarded points is **65.5**

Advice: The number of points on each question is **roughly equal** to the expected amount of time required to answer the question. If you go over the time on a particular question, **move on to the next question**!

Short Table of Laplace Transforms

Function	Laplace Transform
$u(t)$	$1/s$
e^{at}	$1/(s-a)$
t	$1/s^2$
$A\sin(wt)$	$Aw/(s^2+w^2)$
$A\cos(wt)$	$As/(s^2+w^2)$

1. **[3]** The equations below list four different ways to represent a model. Associate words from the following list with each model type. Each model will be associated with three words. Each correct work will yield 1/4 point.

   Linear
   NonLinear
   Discrete
   Continuous
   Time Dependent
   Time Invariant

   For example, for the first equation you might associate the words, Nonlinear, Continuous (which happens to be wrong!).

$$x(k+1) = Ax(k) + Bu(k)$$

$$\frac{dx}{dt} = Ax(t) + Bu(t)$$

$$\frac{dx}{dt} = f(x(t), u(t))$$

$$\frac{dx}{dt} = f(t, x(t), u(t))$$

   A.
   a. discrete, linear, time invariant b. continuous, linear, time invariant c. continuous, nonlinear, time invariant d. continuous, nonlinear, time dependent

2. **[2]** Show that the following equation is nonlinear:

$$y = x^2 + x$$

A. $y_1 = x_1^2 + x_1 \;\; y_2 = x_2^2 + x_2 \;\; y_1 + y_2 = x_1^2 + x_1 + x_2^2 + x_2$ but $(y_1 + y_2) = (x_1 + x_2)^2 + (x_1 + x_2) = x_1^2 + x_2^2 + 2x_1x_2 + x_1 + x_2 \neq y_1 + y_2$ Equation is nonlinear

3. **[1]** Write out and name of the two properties you used to prove linearity in the last question.

A. Additivity $F(x_1 + x_2) = F(x_1) + F(x_2))$ and Homogeneity $F(ax) = aF(x)$

4. **[3]** Are the following statements true or false (1/2 point each)?

a) Linear system can have multiple steady state solutions.
A. False

b) Nonlinear systems can affect the phase, amplitude and frequency of a sinusoidal input signal.
A. True

c) In a linear system the forced and free responses cannot be simply added together to form the total response.
A. False

d) Linear systems generally have simple behavior.
A. True

e) There are no analytical solutions to linear systems.
A. False

f) Nonlinear system have much more interesting dynamics compared to linear systems.
A. True

5. **[2.5]** A pharmaceutical company wishes to build a patient model that involves describing the distribution of a drug through

the body as the drug is given in the form of a rapid intravenous injection. The model comprises three compartments, blood, liver and brain and excretion. First order kinetics governs the movement of drug from one compartment to another. **List five assumptions** that might have been made in building this model.

A.
1. Well stirred compartments
2. Other organs not important
3. Simple one step processes in move material from one compartment to another.
4. Linear laws in determining mass movement
5. The patient is assumed to be still and no eating

6. **[3]** State the state variables, inputs and parameters in the previous model.

   A.
   State variables: concentrations. Parameters: Kinetics constants. Inputs: Injection of drug into patient

7. The pharmaceutical company described in the previous question wishes to solve the differential equations that represent the model. To begin with they choose to use the Euler method to integrate the differential equations.

   a) **[1.5]** Describe the Euler method, an equation, sentence or diagram will suffice in your description

   A. $y(t+h) = y(t) + h dy/dt$

   b) **[1.5]** Do you think they made the right choice? Explain your answer;

   A.
   No, because the Euler method is very sensitive to the step size and requires very small step sizes to work effectively

c) **[1.5]** How would you go about testing whether the solution generated by the Euler method is reasonable?

A. Try two different step sizes to see if the output trajectories are similar, if not reduce the step size further

d) **[1.5]** An overpaid consultant who is an expert in Matlab is brought into the company. The consultant recommends a different ODE solver that is provided by Matlab. Which two solvers might the consultant recommend the company use and why?

A. Use ode45 or ode15s. ode45 implements an adaptive step size RK4/5. ide15s implements a stiff method that is specially designed to handle systems with varying time scales.

8. **[1]** How much might the consultant charge for this service to the company, $200 per hour, $400 per hour or $800 per hour?

9. Newton-Raphson Questions

   a) **[3]** Use a diagram to explain how the Newton-Raphson algorithm works (Use the back of the exam sheet).

   b) **[1]** Write out the equation that describes the scalar version of the Newton-Raphson algorithm.
   A.
   $x_{i+1} = x_i - f(x_i)/f'(x_i)$

10. In solving linear differential equations there are at least two types of solution of interest, these are called the free response and the forced response.

    a) **[1]** What do we mean by the free response?
    A.

It is the response we get when there is no input but the initial conditions are non-zero

b) **[1]** The equation $Y(s) = H(s)U(s)$ refers to which of the two responses?

A.

Forced response

c) **[1]** Identify the free and forced parts in this equation:

$$y(t) = y(0)e^{-kt} + \frac{v_o}{k}\left(1 - e^{-kt}\right)$$

A.

First is free, second part if forced

11. **[1.5]** Define $H(s)$, the transfer function.

A.

A Transfer Function is the ratio of the output of a system to the input of a system, in the Laplace domain give that initial conditions are zero.

12. **[2]** The Laplace transform for an input signal is given by:

$$U(s) = \frac{1}{s^2}$$

and for the output of the system is:

$$Y(s) = \frac{s^2}{1+s}$$

What is the transfer function, $H(s)$ for this system?
A.
$H(s) = \dfrac{s^4}{1+s}$

13. **[5]** One of the key results developed in control theory is that given the transfer function. $H(s)$, the output of a system, $Y(s)$ can be determined for any given input, $U(s)$. However given that $H(s)$ assumes that the initial conditions are zero, $x(0) = 0$, this ability might seem much less interesting. Comment on this statement (Hint: Linearization)
A.
When we linearize we create a new set of variables that are defined as changes relative to the operating point. Since the operating point can be at steady state, it is necessarily so that the absolute initial conditions have to be zero.

14. **[1]** For a system with three state variables, how many transfer function will there be?
A.
3

15. **[2]** For a system with three state variables, comment on what is similar about their transfer functions? Explain your answer
A.

Same denominator (characteristic equation). The denominator comes from the inverse of the A matrix, hence it is the same for every state variable.

16. **[1]** Given a transfer function $H(s) = 1/(1 + s)$, indicate whether this system is stable or not.
    A.
    stable

17. **[1]** Explain the answer you gave in the previous question. An answer such as "Because the root is negative/positive" is insufficient and will be awarded zero points.
    A.
    The solution to a linear system is of the form $\beta e^{\lambda t}$ where λ are the roots of the characteristic equation. If the roots are negative then the exponents will decay to zero, indicating stability.

18. **[2]** The characteristic equation for a two variable system admits a conjugate pair as its solution. The real parts of the pair are positive. What behavior will this system likely have when perturbed near its operating point? Sketch the dynamics on a phase plot.
    A.
    Spiral outwards

19. **[4]** Given the block diagram shown below, find the transfer function $H(s) = C(s)/R(s)$.

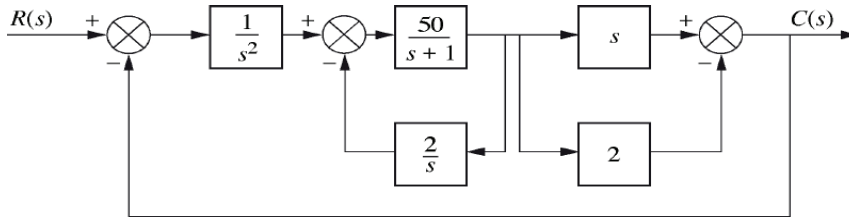


Figure 1: Block diagram

A.

$$H(s) = \frac{50(s-2)}{s^3 + s^2 + 150s - 100}$$

20. Figure 2 shows a Bode plot of the open gain loop for a negative feedback system.

    a) **[1.5]** Mark on the diagram the phase and gain margin

    b) **[1.5]** State whether the feedback system is likely to stable or unstable

    A.
    Unstable

    c) **[2]** State your reasons for your answer in b)

    A.
    At 180 degrees on the phase plot, the gain is greater than one. At unity gain, the phase is greater than 180

21. **[3]** A typical mammalian cell receives many graded external signals via hormonal inputs. These inputs often operate on what appear to be protein networks that behave as negative feedback amplifiers. The output of these negative feedback

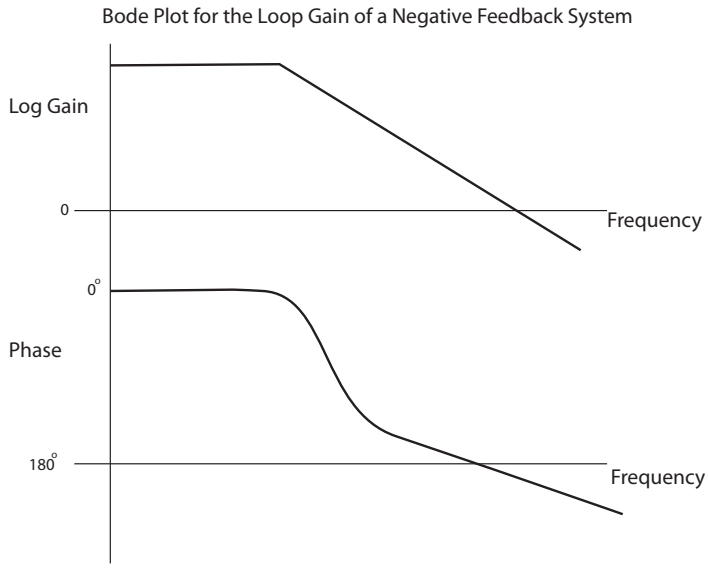


Figure 2: Bode Plot

amplifiers are often used to alter gene expression in the nucleus. List **three** advantages that negative feedback can provide in such a system.

A.
1. Robust to component variation (genetic variability) 2. Maintains linearity when signal is amplified 3. Robust to downstream demand.

22. **[1.5]** Quick fire questions, worth $1/4$ point each

    (a) What does the s in $H(s)$ represent?

    (b) What do the symbols in jw represent?

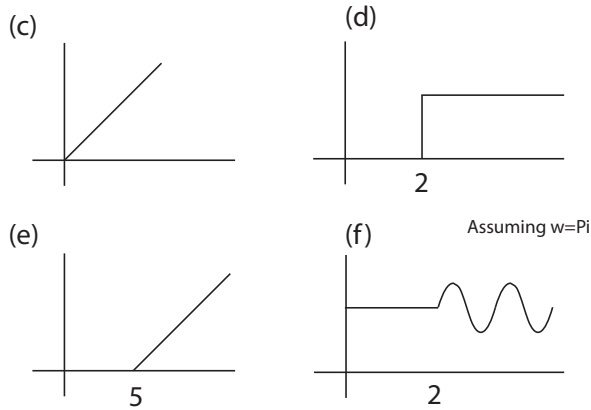
    (c) Sketch the signal $u(t) = 2t$.

    (d) Sketch the signal $u(t-2)$

(e) Sketch the signal $(t-5)u(t-5)$

(f) Sketch the signal $u(t)+\sin(wt)u(t-2)$

A.
a) $\sigma + j\omega$
b) Complex number and frequency



Basic Math Related Questions

23. **[2]** Obtain the Laplace transform for the following equation:

$$f(t) = \left(1 - e^{-at}\right) u(t)$$

A. $\mathcal{L}[f(t)] = \dfrac{a}{s(s+a)}$

24. **[2]** Apply partial fraction decomposition to the following equation:

$$Y(s) = \frac{s+1}{s^3 + 5s^2 + 6s}$$

A.

$$\frac{1}{6s} + \frac{1}{2(s+2)} - \frac{2}{3(s+3)}$$

25. **[2]** Find the inverse transform for the following equation:

$$Y(s) = \frac{1}{5s} + \frac{1}{3(s+2)} - \frac{4s}{(s^2+4)}$$

A.

$$y(t) = \frac{1}{5}u(t) + \frac{1}{3}e^{-2t} - 4\cos(2t)$$

26. **[2]** Use the final-value theorem to determine the value, $y(t)$, of the system at infinite time given the transfer function:

$$Y(s) = \frac{3(s+2)}{s(s^2+s+10)}$$

A.
Use $x(\infty) = \lim_{s \to 0} sX(s)$ therefore $x(\infty) = 0.6$.